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Probing the violation of equivalence principle at a muon storage ring via neutrino oscillation

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Abstract

We examine the possible tests of violation of the gravitational equivalence principle (VEP) at a muon storage ring via neutrino oscillation experiments. If the gravitational interactions of the neutrinos are not diagonal in the flavour basis and the gravitational interaction eigenstates have different couplings to the gravitational field, this leads to the neutrino oscillation. If one starts with μ^+ beam then appearance of $\tau^\pm$, e^+ and μ^- in the final state are the signals for neutrino oscillation. We have estimated the number of μ^- events in this scenario in $\nu_\mu - N$ deep inelastic scattering. Final state lepton energy distribution can be used to distinguish the VEP scenario from the others. A large area of VEP parameter space can be explored at a future muon storage ring facility with moderate beam energy.

1 General Strategy

The oscillation among the different neutrino flavours is now a well accepted solution to the solar neutrino problem [1, 2]. Recent results from the SuperKamiokande (SK) experiment [3] at Japan on atmospheric neutrinos also support this proposition. The most popular explanation behind the neutrino oscillation is that neutrinos have non-degenerate masses and the mass eigenstates are not the same as gauge interaction eigenstates. Oscillation probability from one flavour to another is proportional to Δm^2, where Δm^2 is the difference between the square of the physical masses. Lots of work have already been done in this direction [4].

An alternative to this "mass-mixing (MM) solution" was proposed long back by Gasperini and independently by Halprin and Leung [5]. Their idea was that gravitational interaction of the neutrinos may not be diagonal in the flavour basis. Thus the gravitational interaction eigenstates are different from the weak interaction eigenstates. Now if these gravitational interaction eigenstates couple to the gravitational potential with different strength, neutrino oscillation can take place. This mechanism does not require neutrinos to have non-zero masses.

The principle of equivalence has been the cornerstone of the general theory of relativity. Validity of this principle has been tested to a high precision for macroscopic bodies. But this has

not been tested experimentally in the microscopic and quantum regime. So it is prudent to keep an open mind towards this issue and try to prove/disprove the validity of equivalence principle in the current and future experiments.

For a two flavour oscillation picture[1], the most general expression for the transition probability is given by,

$$P_{i \leftrightarrow j} = \sin^2 2\theta \ \sin^2 \left(\frac{\pi L}{\lambda} \right) \tag{1}$$

L, is the base-line length (distance which neutrinos traverse from the source to the detector). θ is the usual mixing angle.

Expressions for λ for the two cases (MM and VEP) are,

$$\begin{aligned} \lambda &= \frac{4\pi E_\nu}{\Delta m^2} \ \ (MM) \\ &= \frac{2\pi}{E_\nu \phi \Delta f} \ \ (VEP) \end{aligned} \tag{2}$$

In the above equations, E_ν is the neutrino energy. Δf ($\equiv f_1 - f_2$) represents the difference between the coupling strengths of the two gravitational eigenstates with gravitational potential ϕ and it quantifies the violation of equivalence principle in the neutrino sector. The basic difference between the two cases, on the dependence of E_ν, is evident. So their predictions can be different altogether. This alternative has attracted many people and a lot of work has been done [6]. Oscillation probability induced by VEP is,

$$P = \sin^2 2\theta \ \sin^2 \left(2.538 \times 10^{18} \ L E_\nu \Delta f \phi \right) \tag{3}$$

Here, L is expressed in kilometres and E_ν in GeV. We will assume that ϕ remains unchanged over the neutrino path. But, this is not very crucial for our purpose, we will parametrise VEP by the product $\phi \ \Delta f$ ($\equiv \Delta F$). Some recent work [7], analysing the SK solar neutrino data, gives the best fit value for this parameter. This comes out to be as small as 10^{-24} for maximal mixing.

In this letter, we will try to explore the possible signals of VEP at a muon storage ring. Here, we assume the sample design, for muon production, capture, cooling, acceleration and storage as given in ref. [8]. Number of available muons directed to the neutrino detector of per year is 10^{20} (one can have μ^+ or μ^-) . For the purpose of illustration, we start with μ^+ beam. These positively charged muons will produce $\sim 10^{20}$ $\bar{\nu}_\mu$s and same number of ν_es. Muon anti-neutrinos (or electron neutrinos) thus produced, traverse a distance L before colliding on a fixed target. For a sufficiently energetic initial muon beam, neutrino-nucleon (in the target material) interaction is in the deeply-inelastic regime. Now, on their way to the detector, if some $\bar{\nu}_\mu$s are oscillated to $\bar{\nu}_e$ or $\bar{\nu}_\tau$s, in the final state we may observe τs or es with the same sign with the initial muon beam, via neutrino nucleon charged current interaction. The ν_es (also coming from μ decay) may transform to ν_μs or ν_τs, which can be the source of μ^-s or τ^-s. We will call them wrong sign μs or τs, as they have opposite charge to the initial μ-beam. The un-oscillated $\bar{\nu}_\mu$ (ν_e)s can only

[1] we will stick to this in the present analysis due to simplicity. But this is sufficient to illuminate the underlying principle.

give rise to μ^+ (e^-)s. So appearance of τ leptons of either sign and negatively charged muons or positively charged electrons are definitely the signal for neutrino oscillation at a muon storage ring with positively charged muon-beam. These issues have been studied in detail earlier [9] in the context of MM scenario. Here we will mainly concentrate on the results for μ^- appearance in the final state. This means our focus is on the $\nu_e \leftrightarrow \nu_\mu$ oscillation due to VEP. We will see in the following, that μ^- event rate in MM scenario is negligibly small with presently SK allowed values for $\Delta m^2_{e\mu}$ and $\sin^2 2\theta$. So appearance of considerable number of μ^- events at a muon storage ring, is a signature which goes against the MM solution to neutrino oscillation problem. So the charge identification for the muons is necessary for the study of neutrino oscillation physics. This seemed to be achievable according to the ref.[8]. VEP solution does not fit very well to the $\nu_\mu - \nu_\tau$ oscillation data from SK. So we will not calculate the τ^+ appearence rate in the following.

For lower values of energy of the initial muon beam, τ^- cross-section is slightly lower than the μ^- case due to the phase space suppression. Furthermore, for tau in the final state, above cross-section has to be multiplied by appropriate branching ratio of τ. Generally one prong hadronic or leptonic decay channel is the cleanest way to detect a τ. Former has a branching ratio of 45 % whereas the later has of 17 %.

Number of μ^- events coming from neutrino-nucleon DIS can be obtained by folding the charged current cross-section by oscillation probability, neutrino flux and finally by the total number of nucleons, N_n, present in the fixed target. This last quantity depends on the amount of the target material present.

$$N_\mu = N_n \int \sigma(\nu_\mu + N \rightarrow \mu^- + X) \, \frac{dN_\nu}{dE_{\nu_e}} \, P(\nu_e \leftrightarrow \nu_\mu) \, dE_{\nu_e} \tag{4}$$

We present our results for 1 kT of the target material[2]. For any other detector size these results can easily be scaled. The charged-current neutrino-nucleon cross-section is calculated assuming almost the same number of protons and neutrons, present in the target material. The expression for the same is not given here but can be obtained in several places [8, 9]. Our results are based on a simple parton level monte-carlo event generator. We have not incorporated any detector effects. Even incorporating these, the essence of our analysis will remain the same. We have used CTEQ4LQ parametrisations [10] for the parton distribution functions to estimate the above cross-section.

Neutrino flux depends on the number of neutrinos, initially present. At a muon storage ring this number is equal to the number of muons present in the beam. Neutrino flux also depends on energy/angular distribution of the neutrinos coming from μ decay. We will elaborate more on this in the following.

The area of the target and the base-line length, L, define a cone with half angle θ_d, which can be written as $R_d = L\ \theta_d$. (R_d is the radius of the detector if we assume it has a circular cross-section.) Thus the choice of detector size for a long-baseline experiment must differ with that of a short base-line experiment and also depends on the energy of the muon beam. If the muon-beam

[2]This corresponds to $N_n = 6.023 \times 10^{32}$

energy is increased, neutrinos coming from muon decay, become more and more boosted in the muon direction. This effectively increases the luminosity of neutrino beam. A higher effective luminosity of the incident neutrino beam can be achieved with a sufficiently high energy beam and/or by decreasing the effective area of the target (keeping the amount of target material fixed). In our event generator, we only consider those νs, for which $\theta_{\mu,\nu} < \theta_d$. ($\theta_{\mu,\nu}$ is the angle between the decaying muon beam and the neutrino.) Of course this is fully determined by the muon decay kinematics. In this analysis we will assume a fixed detector size with $R_d \sim 6$ *mts.*. Target area corresponding to this radius are similar to that of proposed ICANOE detector [11].

2 Discussion of the Results

In Fig. 1, we present variation of the number of wrong sign muon events coming from ν_μ-N DIS experiments for VEP scenario. Number of tau events, from ν_τ-N scattering will almost be the same as this. We have chosen $\Delta F = 2{\times}10^{-24}$ and $\sin^2 2\theta$ as 1. Choice of these parameter values are from ref. [7]. They have used the SK solar neutrino data, to calculate the best fit values of ΔF and $\sin^2 2\theta$. The only other bounds on these two parameters come from the LSND and E776 experiment at BNL [12]. Ref. [7] constrains ΔF responsible for the $\nu_e \leftrightarrow \nu_x$ ($x \equiv \mu, \tau$ *or stetrile*) oscillation. The other one [12] constrains the same parameters for $\nu_e \leftrightarrow \nu_\mu$ oscillation.

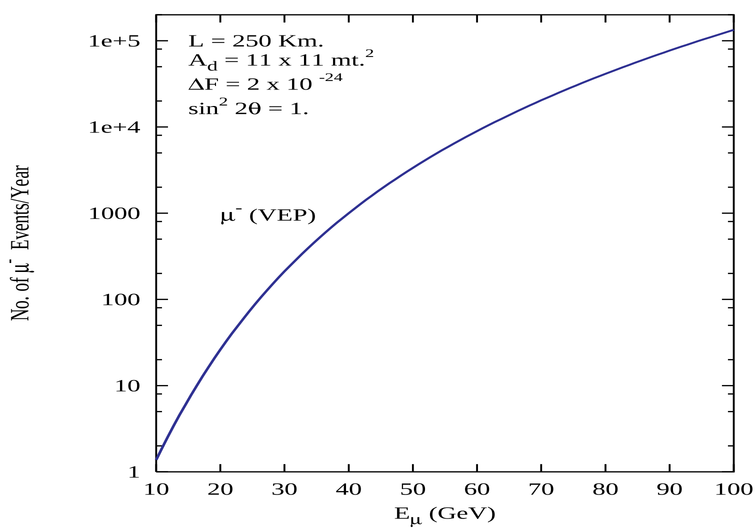

Figure 1: *Number of* μ^- *events coming from* ν_μ*-N DIS at a muon storage ring with* $\Delta F = 2 \times 10^{-24}$ *and* $\sin^2 2\theta = 1$. A_d *is the area of the target.*

We will not present here the τ^+ event rate from $\bar{\nu}_\tau - N$ scattering. VEP solution to atmospheric $\nu_\mu - \nu_\tau$ oscillation is disfavoured by the the SK data [14, 15, 16]. In ref. [15, 16], a χ^2 analysis has been done with SK atmospheric neutrino data, assuming a general power law dependence of oscillation probability on the neutrino energy ($\sim E^n$). Minimum χ^2 is obtained for $n = -1$ *i.e.* for the MM scenario.

But it should be borne in mind that along with this indirect evidence (χ^2 analysis in the ref. [15, 16]) against the VEP solution to the atmospheric $\nu_\mu - \nu_\tau$ oscillation, one also should look for

some direct evidences to disprove such propositions. Neutrino factory with a huge and precisely known neutrino flux is an ideal place for this. $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_\tau$ oscillation due to VEP and subsequent $\bar{\nu}_\tau - N$ scattering can produce τ^+ events. The search for τ appearence in final state will be one of the major goals of neutrino oscillation experiments. So we would not have to pay some extra price to look for τ^+ signal which can can arise from VEP in neutrino sector.

We have checked that event rates remain almost unchanged for two baseline lengths (First one, 250 Km., corresponds to a baseline from Kamioka to KEK and the second one, 740 Km., corresponds to a baseline from CERN to Gran Sasso or from Fermilab to Soudan experiment.) over the energy range we used in fig. 1. For the values of neutrino energy and ΔF we have used, oscillation probability grows quadratically with L. At the same time, for a fixed target area and size, effective luminosity of colliding neutrinos decreases with L, due to the decrement of the detection cone subtended by the target. These two effects compensate each other, for the detector area we have chosen. We checked that for a sufficiently large target area, and/or sufficiently high muon beam energy, when almost all the neutrinos coming from muon decay can be intercepted by the target, (i.e. when neutrino flux becomes independent to L) the event rate indeed grows quadratically with L.

Number of μ^- events in MM scenario, when calculated using SK allowed values for mass^2 difference and mixing angle ($\Delta m^2_{e\mu} = 2 \times 10^{-5} eV^2$, $\sin^2 2\theta = 1$. These values are for MSW solution. Vacuum solution gives $\Delta m^2_{e\mu} \sim 10^{-10} eV^2$ [18]), comes out to be small compared to the VEP scenario over the range of muon energies we are interested and is beyond the scale used in fig.1. The other source of μ^- is from the τ^- (produced in $\nu_\tau - N$ interaction) decay. But this rate is further suppressed by $\tau \to \mu$ branching ratio. Thus the appearance of μ^- in final state (starting with μ^+ beam), is a definite signature of neutrino oscillation not originated by MM scenario. Appearance of μ^- in the final state may also be accounted by the R-parity violating μ-decay and/or $\nu - N$ interaction [19][3].

We confine our discussion to the appearance of wrong sign muons in the final state. Wrong sign τ event rate will be somewhat lower (due to phase-space suppression) than this if difference of couplings between ν_μ and ν_τ to the gravitational potential is at the same ballpark of ν_μ and ν_e. Apart from ref. [7], the only bound on the ΔF, coming from two terrestrial experiments (LSND and E776) are not consistent with each other. And these values are order of magnitude greater than the best fit value obtained in ref. [7]. Upper bound coming from the E776 experiment is around 3×10^{-21} for $\sin^2 2\theta = 1$. On the other hand LSND results predicts a lower bound on ΔF ($\sim 5 \times 10^{-18}$) for the same value of the mixing angle. The SK situation is a bit different from the other two cases (In terrestrial experiments, neutrinos are mainly affected by the earth's gravitational field. In the former, neutrinos traverse the sun-earth distance. Gravitational fields of sun and near by stars along with the earth have to be considered in such a case.). We want to stress that even with such a small value of the above parameter, number of μ^- events in VEP scenario will be much larger than that of MM scenario. In the following we will explore the

[3]R-parity violating theories allow lepton flavour violation, thus μ^- or τ appearance in such a theory can also be explained without flavour oscillation.

possible region in $\Delta F - \sin^2 2\theta$ space for which a perceptible signal can be obtained such that we can exclude that region at a certain confidence level.

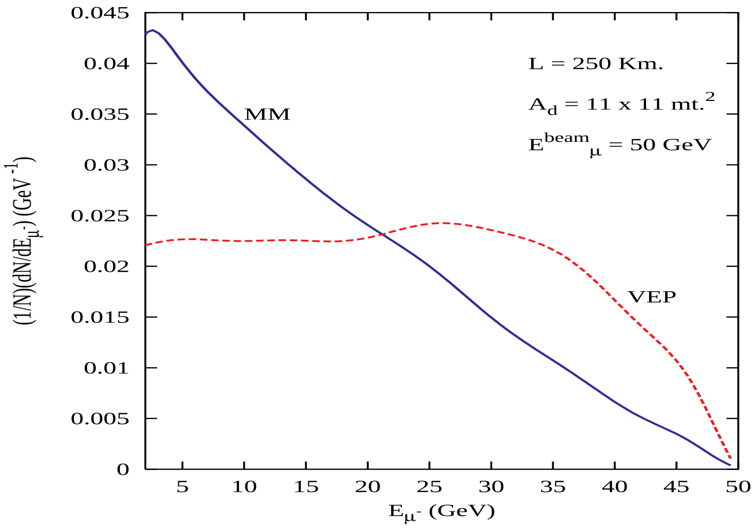

Figure 2: *Normalised μ^- energy distribution at a 50 GeV muon storage ring. Solid curve is for MM scenario and the dashed curve is for VEP scenario A_d is the target area.*

Before we go into the exclusion contours, we want to point out another distinguishing feature of the VEP scenario, which is also an artefact of the energy dependence of oscillation probability. We look at the normalised energy distribution of the μ^- in the final state. In MM scenario lepton energy distribution falls of rapidly in contrast to the VEP scenario. This is reflected in fig. 2. If the VEP parameter ΔF is so small that event rate is comparable with event rate in the MM scenario [4], energy distribution of the final state lepton can be used to discriminate between these two scenarios. So one can have an idea about the energy dependence of the oscillation probability from the lepton energy distribution. If one concentrates on the μ^- appearance rate, then MM scenario is outnumbered by VEP scenario. Strong constraint can be put on this scenario from the wrong sign muon search. We have already mentioned about μ^- or τ^+ appearance due to the R-parity violating interaction at a muon storage ring. Wrong sign muon or tau events can be accounted without the oscillation phenomena in such a kind of theories. Here also the lepton energy distribution is expected to be different from MM scenario.

Until now, we have used the best fit values for ΔF and $\sin^2 2\theta$ as given in ref. [7], but the main purpose of this letter is to point out that even a tiny violation of equivalence principle can be detected at a muon storage ring facility and can be distinguished from the MM scenario quite efficiently. In the following we like to show, in the $\Delta F - \sin^2 2\theta$ plane, the region which can be excluded at 95 % C.L.

From our previous discussions, it is clear that μ^- appearance search is the best probe for VEP at a neutrino factory. Firstly, μ^- event rate is higher than the τ^- event rate. And with

[4]One can see from previous discussion, μ^- event rate in MM scenario is negligibly small compared to the same in VEP scenario with the values of parameters we have used. But having larger amount of target material and at the same time smaller value of ΔF can make these two comparable with each other.

experimentally allowed values of MM parameters, one cannot have μ^- signal from MM scenario. We will present the exclusion contours from μ^- appearance search for two values of beam energy.

We will not discuss here in details the possible backgrounds for such signals. These issues have been discussed in ref. [20]. We have applied the same kind of cuts to reduce the background.

$$\begin{aligned} p_T^{\mu^-} &> 2\ GeV, \\ \Delta R_{\mu^- X} &> 0.4 \end{aligned} \tag{5}$$

$\Delta R_{\mu^- X}$ is the isolation between the μ^- and the DIS hadronic products. After applying the above set of cuts the signal efficiency comes out to be 70 %. The other source of wrong sign muon is from the decay of the wrong sign taus' coming from the $\nu_\tau - N$ scattering. These ν_τs come from $\nu_e \leftrightarrow \nu_\tau$ oscillation. So this is also due to flavour oscillation and add to our signal. But we confine ourselves to two flavour oscillation only and do not add the numbers of wrong sign muons coming from tau decay.

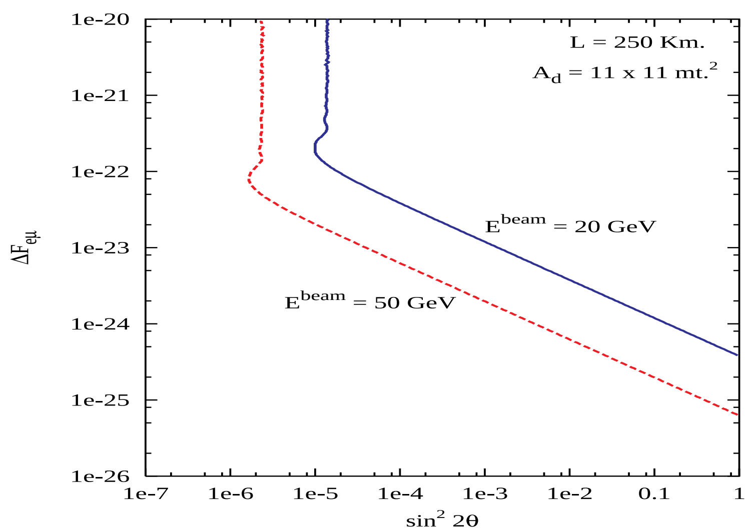

Figure 3: *95 % exclusion contour in $\Delta F_{e\mu} - \sin^2 2\theta$ plane for 50 and 20 GeV muon storage ring from wrong sign muon signal.*

The exclusion contour depicts the fact that for small values of ΔF one need maximal mixing between the flavours. For higher values of VEP parameter very small mixing would sufficient to produce enough number of signal events. For ΔF less than 10^{-23} , $\sin^2 2\theta$ and ΔF are correlated in a linear fashion. In this region, oscillation wave-length λ (see eqn. 1) is much larger than the base-line length, L. For larger values of ΔF, L is comparable with λ. Here, $\sin^2(\Delta F\, E_\nu\, L)$ attains its maximum value. This explains the correlation between the two oscillation parameters near the 'knees' of the contours. For even higher values of ΔF, $L >> \lambda$ and the $\sin^2(\frac{\pi L}{\lambda})$ can be approximated by $\frac{1}{2}$. This causes the sharp rise of the contour almost independent of mixing angle.

A different phenomenon which also cause the neutrino oscillation with the same neutrino energy dependence of oscillation probability, is the violation of special relativity (VSR) [21]. One can parametrised the oscillation probability in the same way. So the above analysis is completely

applicable to VSR also. The energy distribution and exclusion contours derived above would be the same for the later.

3 Conclusions

To summarise, we have shown that at a muon storage ring with moderate energy, neutrino flavour oscillation due to VEP can be probed. The μ^- appearance event rate due to $\nu_\mu - N$ DIS has been calculated within RG improved parton model. We compared the VEP results with the MM results and have shown that, the former will give overwhelmingly large number of wrong sign muon events than the later. In VEP model, the number of μ^- events increases with beam energy. The energy distributions of the final state muon have also been compared for two different scenario. If the violation of equivalence principle is so small that it would produce number of events comparable with that of mass mixing model, then energy distribution of the final state lepton can be used as a discriminator. Finally we have shown the 95 % C.L. exclusion contours in the $\Delta F_{e\mu} - \sin^2 2\theta$ plane for two different muon beam energy.

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